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# SYRACUSE UNIVERSITY RESEARCH CORPORATION

Steady-State and Transient  
Performance of the  
Sidelobe Canceller (U)

By

S. P. Applebaum

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STEADY-STATE AND TRANSIENT PERFORMANCE  
OF THE SIDELobe CANCELLER(U)

By

S. P. Applebaum

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## ABSTRACT

(S) The purpose of this study was to determine the factors and trade-offs that limit the speed of response ("lock-on" rate) of the Sidelobe Canceller. The Sidelobe Canceller consists of a correlator, (multiplier and integrator), within a closed loop circuit. The lock-on rate is essentially determined by the bandwidth of the integrating circuit, the loop gain, and the bandwidth of the input jammer waveform. In a Sidelobe Canceller with a linear response in the auxiliary channel, the loop gain increases with the amount of jamming power in the auxiliary channel. The loop gain is determined by the "quiescent" loop gain, (a design parameter equal to the loop gain when only receiver noise is present in the auxiliary channel), and the jamming-to-noise ratio in the auxiliary channel.

(U) The performance of the Sidelobe Canceller, both transient and steady-state can be related to two parameters, the loop gain and the ratio of the enhanced loop bandwidth to the jammer bandwidth,  $B_E/B_J$ . The enhanced loop bandwidth,  $B_E$ , is defined as  $(1 + \text{loop gain})$  times the bandwidth of the integrating circuit.

(S) The lock-on rate can be increased by increasing loop gain and/or the enhanced bandwidth. The loop gain is limited by stability requirements to 30-40 db. The enhanced bandwidth can't be increased much beyond the point where the ratio  $B_E/B_J$  exceeds unity without hurting the steady-state performance.

(S) When the loop response is slow compared to the bandwidth of the jammer, that is, if  $B_E/B_J \ll 1$ , then the transient response is a simple exponential and lock-on occurs at the rate of  $27 B_E$  db per second. If the enhanced bandwidth is increased by increasing loop gain and/or the integrating circuit bandwidth, the transient response speeds up but the improvement comes at a decreasing rate after the ratio  $B_E/B_J$  exceeds unity. When  $B_E/B_J$  is unity the lock-on rate is  $16 B_E$  db per second.



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(S) When the ratio of  $B_E/B_J$  is increased beyond unity. The steady-state cancellation performance deteriorates. The amount of jammer cancellation is reduced and desired signal will also be cancelled somewhat. (The signal is assumed to have a narrow bandwidth compared to the jammer bandwidth.) There will also be some cancellation of receiver noise components but not quite as much. Thus there is loss in signal to noise ratio as well as a change in signal level. The loss in detectability will be equal to the drop in signal level unless some form of AGC or CFAR is used.

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## I. Introduction

(S) Sidelobe cancellation is an ECCM technique designed to cope with the screening sidelobe jammer. The first level of defense against the sidelobe jammer clearly lies in the design and control of the antenna pattern. However, sidelobe levels achievable in practice are limited by the stability of components, effects of environmental fluctuations, the sensitivity and response time of any monitoring system, and by site conditions. Even under ideal conditions, the design sidelobe level cannot be made arbitrarily small because of the resultant increase in main beamwidth and loss in gain.

(S) Sidelobe cancellation may be regarded as an adaptive technique for sidelobe control. The sidelobe canceller uses the jamming signal to modify the radar antenna response pattern so as to produce a near-null in the direction of the jammer. Strong jamming signals can be "nulled" better than weak jamming signals; as a result the jammer residue after cancellation tends to be independent of jammer strength.

(S) One Sidelobe Canceller using the signal from an appropriate auxiliary sensor can cancel one jammer over a narrow bandwidth. In general, multiple Sidelobe Cancellers are required to cancel multiple jammers or a single jammer over a wide bandwidth. Both single and multiple jammer cancellers have been developed and demonstrated in the field on narrowband, air-defense, surveillance radars.

(S) In a defense environment that is changing slowly with time, the "steady - state" performance of the canceller is of main concern. However, if the jammer configuration is changing rapidly with time or if the antenna beam must be switched rapidly from one direction to another, then the cancellation loops must respond rapidly and "transient" performance becomes important. The purpose of this report is to investigate the transient response of a single canceller loop and find what trade-offs are involved between designing for fast lock-on time and steady-state cancellation performance.

## II. "Available" Steady-State Performance

(U) In order to have a basis for evaluating the steady-state performance of a canceller, we will determine first the amount of cancellation achievable by an optimum canceller. Consider the situation shown in Figure 1. Channel "M" is the main information bearing channel containing both the desired signal and a jamming signal. Channel "A" is an auxiliary channel that contains a component correlated with the jamming signal in the main channel. We assume that the auxiliary channel contains at most a negligible amount of the desired signal in the main channel. The object is to improve the ratio of desired signal to jamming signal in the main channel by subtracting from it the signal in the auxiliary channel after appropriate amplitude and phase weighting.

(U) In Figure 1,  $s(t)$  is the complex envelope of the desired signal,  $v_m(t)$  is the complex envelope of the jamming signal plus receiver noise in the main channel,  $v_a(t)$  represents jamming signal and receiver noise in the auxiliary channel. Since the amount of desired signal in the main channel is not affected by the subtraction, signal-to-noise ratio will be optimized by minimizing  $r(t)$ , the jamming plus receiver noise residue in the output.

(U) As shown in Figure 1, the auxiliary channel signal is multiplied by the complex weight "x" and the result subtracted from the main channel, leaving a residue of

$$r(t) = v_m(t) - x v_a(t) \quad (1)$$

(U) Without the auxiliary channel signal, ( $x = 0$ ), the expected residue power<sup>(1)</sup> would be,

$$\overline{|r_o(t)|^2} = \overline{|v_m(t)|^2} \quad (2)$$

---

<sup>(1)</sup> Throughout this report "power" refers to envelope power which is twice the actual power in the real signal.

whereas with the cancellation signal

$$\overline{|r(t)|^2} = \overline{|v_m|^2} - x \overline{v_a v_m^*} - x^* \overline{v_a^* v_m} + |x|^2 \overline{|v_a|^2} \quad (3)$$

$$= \overline{|v_m|^2} - \frac{|\overline{v_m v_a^*}|^2}{\overline{|v_a|^2}} + \overline{|v_a|^2} \left| x - \frac{\overline{v_m v_a^*}}{\overline{|v_a|^2}} \right|^2 \quad (4)$$

(Note: "\*" denotes complex conjugate)

(U) It is easily seen from (4) that the value of  $x$  that minimizes the expected residue power is given by,

$$x_{\text{opt}} = \frac{\overline{v_m v_a^*}}{\overline{|v_a|^2}} \quad (5)$$

With this value of  $x$ , the residue power becomes,

$$\overline{|r_{\text{min}}|^2} = \overline{|v_m|^2} - \frac{|\overline{v_m v_a^*}|^2}{\overline{|v_a|^2}} \quad (6)$$

$$= \overline{|v_m|^2} \left\{ 1 - \frac{|\overline{v_m v_a^*}|^2}{\overline{|v_m|^2} \overline{|v_a|^2}} \right\} \quad (7)$$

$$= \overline{|v_m|^2} \left\{ 1 - |\rho_{ma}|^2 \right\} \quad (8)$$

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where  $\rho_{ma}$  = correlation coefficient of  $v_m$  and  $v_a$

(U) The cancellation ratio, which is defined as the ratio of the residue with cancellation to the residue without cancellation, is given by.

$$\text{cancellation ratio} = \frac{\overline{|r|}^2_{\min}}{\overline{|r_o|}^2} = 1 - |\rho_{ma}|^2 \quad (9)$$

(S) From this we see that the amount of cancellation achievable depends only upon the magnitude of the correlation coefficient. This in turn is fundamentally limited by receiver noise, spatial separation of the main and auxiliary antennas, spatial distribution of the jammers, and their bandwidths. As an example consider the case where there is one narrow-band jammer present. The signals in the main and auxiliary channels could then be represented as

$$v_m(t) = C J(t) + n_m(t) \quad (10)$$

$$\text{and} \quad v_a(t) = J(t) + n_a(t) \quad (11)$$

where  $J(t) = CE^{(2)}$  of the jamming waveform in the auxiliary

$n_m(t) = CE$  of receiver noise in the main channel

<sup>(2)</sup> COMPLEX ENVELOPE

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$n_a(t)$  = CE of receiver noise in the auxiliary channel

and  $C$  = complex constant

(S) The correlation coefficient between the main and auxiliary signals is given by,

$$\begin{aligned} |\rho_{ma}|^2 &= \frac{|\overline{v_m v_a^*}|^2}{|\overline{v_m}|^2 |\overline{v_a}|^2} \\ &= \frac{|C|^2 P_J^2}{(|C|^2 P_J + N_m)(P_J + N_a)} \end{aligned} \quad (12)$$

where  $P_J = |J|^2$  = auxiliary jammer power

$N_m$  = main receiver noise power

$N_a$  = auxiliary receiver noise power

(S) The residue power, from equations (8) and (12) will therefore be,

$$\begin{aligned} \overline{|\tau|^2} &= (|C|^2 P_J + N_m) \left( 1 - \frac{|C|^2 P_J^2}{(|C|^2 P_J + N_m)(P_J + N_a)} \right) \\ &= N_m + \frac{|C|^2 P_J}{1 + P_J/N_a} \end{aligned} \quad (13)$$

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(S) From this we see that the receiver noise component of the main channel is unchanged but the jammer power is reduced approximately by the jammer-to-noise ratio in the auxiliary. As the jammer power level is increased from zero, the residue power increases monotonically from the noise level to a maximum of,

$$\overline{|r|^2}_{\max} = N_m \left( 1 + \frac{|C|^2 P_J / N_m}{P_J / N_a} \right) \quad (14)$$

(S) The second term in brackets is the ratio of the jammer-to-noise ratios in the main and auxiliary channels, respectively. This ratio will be less than unity if the gain of the auxiliary sensor exceeds the sidelobe gain of the main sensor in the direction of the jammer. In that case the maximum residue will be 3 db above the noise level in the main channel.

(S) When there are more than one jammer present, the correlation between the main and auxiliary channels will in general decrease and the cancellation will be reduced. For example, if there are  $m$  equal power jammers, randomly distributed in space, the mean square correlation averaged over all possible spatial configurations will be  $1/m$ . The cancellation ratio will therefore be  $(m-1)/m$ . In this situation the canceller in effect cancels the equivalent of only one of the  $m$  jammers. To cancel multiple jammers requires the use of multiple cancellers, with at least one canceller per jammer.

(S) The cancellation of wide-bandwidth jamming signals is more difficult than the cancellation of narrow-bandwidth signals. The fundamental limitation is the reduction in correlation between the main and auxiliary signals due to the spacing of their antennas. With one jammer and one canceller the output residue for small spacing is given approximately by,



$$\overline{|r|^2} \approx N_m + |C|^2 P_J \left\{ \frac{1 + 2\pi B_J \tau \frac{P_J}{N_a}}{1 + \frac{P_J}{N_a}} \right\} \quad (15)$$

where  $B_J$  = jammer signal bandwidth (assumed correlation function is  $e^{-\pi B_J \tau}$ )

and  $\tau$  = difference in arrival times of jammer waveform at main and auxiliary antennas

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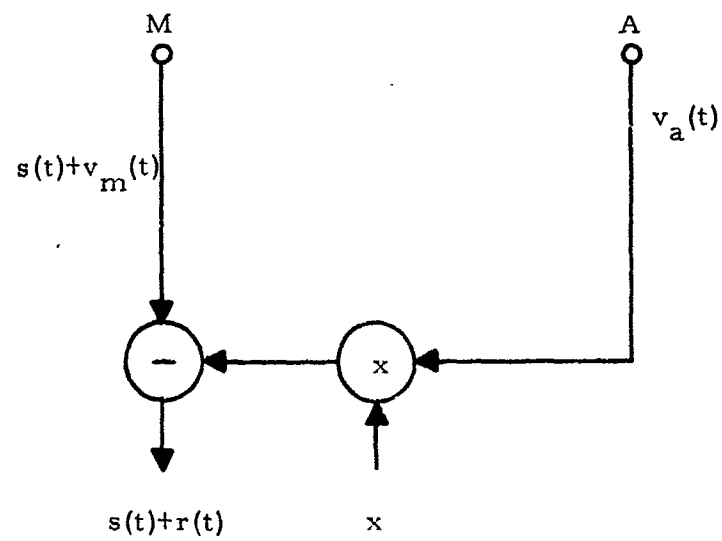


FIGURE 1 -- CANCELLATION PRINCIPLE

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### III. Description of Sidelobe Canceller

(U) The previous section discussed the optimum achievable steady-state performance of an ideal coherent canceller and some of the limitations. In this section we present a mathematical description of an implementable sidelobe canceller. A simplified schematic of an I.F. sidelobe canceller is shown in Figure 2. The signals at various points in the diagram are represented in complex notation. The actual physical signal is simply the real part of its complex representation.

(S) The notation used in Figure 2 is as follows:

$v_m(t)$  = CE of the composite signal in the main channel

$v_a(t)$  = CE of the composite signal in the auxiliary channel

and  $x(t)$  = CE of the output of the integrating filter

(S) As shown in Figure 2 the I.F. carrier frequency,  $\omega_c$ , of the main channel is increased  $\Delta$  radians per second by an up-converter. The auxiliary channel signal,  $v_a(t) \exp j\omega_c t$ , is multiplied by the output of the integrating filter,  $x(t) \exp j\Delta t$ . The upper sideband of this product,  $v_a(t) x(t) \exp j(\omega_c + \Delta) t$ , is subtracted from the signal in the main channel to form the output of the SLC. The complex envelope of the SLC output is therefore,

$$r(t) = v_m(t) - v_a(t) x(t) \quad (16)$$

(S) This output signal is multiplied by the signal in the auxiliary channel and the lower sideband of this product,  $r(t) v_a^*(t) e^{j\Delta t}$ , is fed to the integrating filter. This filter is a high-Q single-pole, band pass filter centered on the radian frequency  $\Delta$ . The output of this filter, as previously noted, is  $x(t) \exp j\Delta t$ .

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(S) Since the bandwidth,  $B_I$ , of the integrating filter is very small compared to its center frequency, its response to an I. F. signal is analogous to that of an RC filter operating on a low-pass signal. We can therefore write the following differential equation for  $x(t)$ .

$$T \frac{dx}{dt} + x = A r(t) v_a^*(t) \quad (17)$$

where  $T = 1/\pi B_I$  = time constant of the integrating filter

and  $A$  = gain factor

(S) Substituting equation (16) into (17) we obtain the basic differential equation of the SLC.

$$T \frac{dx}{dt} + \left\{ 1 + A |v_a(t)|^2 \right\} x = A v_m(t) v_a^*(t) \quad (18)$$

(S) This equation expresses the first-order behavior of the SLC. It neglects the effects of other filters in the circuitry which must be included for sideband selection, amplification, etc. These circuits influence the ultimate stability of the loop and hence limit the amount of loop gain achievable, but with proper design have only second order effects on the cancellation performance within the operating region of the loop.

(S) As an aid to understanding, an equivalent circuit for the control voltage,  $x$ , in a standard feedback loop, is shown in Figure 3. The transfer function in the forward path is that of a simple RC amplifier. The feedback gain is equal to the power level  $|v_a|^2$  in the auxiliary channel and the loop gain is  $A|v_a|^2$ .

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(S) Equation (18) is the differential equation that determines the control voltage  $x(t)$ . Equation (16) describes the output or residue of the canceller. From these two equations we note that an increase in  $v_m(t)$ , the signal in the main channel, will produce a corresponding increase in both the control voltage,  $x(t)$ , and the residue,  $r(t)$ . Hence a change in gain in the main channel ahead of the SLC is equivalent to a corresponding change in gain after the SLC and does not affect the transient or steady-state performance. (This assumes that the internal noise introduced by the SLC circuitry is negligible compared to the signal level). On the other hand, the gain in the auxiliary channel ahead of the SLC has a marked effect on the SLC performance. From equation (18) we see that the power level in the auxiliary channel determines the control loop gain. It will, therefore, affect both the transient response and the steady-state performance of the SLC.

(S) In order to simplify the analysis of the SLC we assume that the power spectra of the random signals in the main and auxiliary channels are identical. This assumption applies in the situation where the power spectra of the front end receiver noise and jamming signals in both main and auxiliary channels are determined by the R.F. and I.F. circuits preceding the SLC. This assumption permits us to represent the signals in the main and auxiliary channels in terms of two uncorrelated waveforms, as follows,

$$v_a(t) = v(t) \quad (19)$$

$$\text{and} \quad v_m(t) = m v(t) + n(t) \quad (20)$$

(S) Here  $v(t)$  and  $n(t)$  are independent normal noise processes having the same power spectrum shapes with

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$$\overline{|v(t)|^2} = \overline{|v_a(t)|^2} \quad (21)$$

$$m = \frac{\overline{v_m(t) v_a^*(t)}}{\overline{|v_a(t)|^2}} \quad (22)$$

$$\text{and } |n(t)|^2 = |v_m(t)|^2 - |m|^2 |v_a(t)|^2 \quad (23)$$

(S) For convenience we refer to the representation indicated in equations (19) and (20) as the "canonical" representation. In most applications the signals in the main and auxiliary channels are specified in terms of the jamming and receiver noise power levels. This corresponds to a representation of the form

$$v_a(t) = J(t) + n_a(t) \quad (24)$$

$$\text{and, } v_m(t) = CJ(t) + n_m(t) \quad (25)$$

(S) The canonical representation corresponding to the above is easily obtained from equations (21)-(23) as follows,

$$|v(t)|^2 = P_J + N_a \quad (26)$$

$$m = \frac{CP_J}{P_J + N_a} \quad (27)$$

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$$\begin{aligned} \text{and, } |n|^2 &= |C|^2 P_J + N_m - \frac{|C|^2 P_J}{(P_J + N_a)^2} (P_J + N_a) \\ &= N_m + \frac{|C|^2 P_J}{1 + P_J/N_a} \end{aligned} \quad (28)$$

(S) Comparing equation (28) with equation (13) we see that  $n(t)$  represents the minimum residue achievable by an optimum canceller. This is also clear from the definition of  $n(t)$  in the canonical representation.

(S) If the canonical representation is introduced into the differential equation for the control loop, equation (18), we obtain,

$$T \frac{dx}{dt} + (1 + A|v(t)|^2) x = A m v(t) + n(t) \Big\} v^*(t) \quad (29)$$

(S) Since the optimum value of the control voltage is "m", it is convenient to make the substitution,

$$x(t) = m(1 - y(t)) \quad (30)$$

in equation (29). This gives,

$$T \frac{dy}{dt} + (1 + A|v(t)|^2) y = 1 - \frac{Anv^*}{m} \quad (31)$$

(S) The output residue of the SLC can also be expressed in terms

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of the variable  $y(t)$ . Using (30) we get

$$\begin{aligned}
 r(t) &= v_m(t) - x(t) v_a(t) \\
 &= m v(t) + n(t) - m(1-y(t)) v(t) \\
 &= n(t) + y(t) m v(t)
 \end{aligned}
 \tag{32}$$

(S) The mean-square output residue is therefore given by

$$\begin{aligned}
 E \left\{ |r(t)|^2 \right\} &= \overline{|n(t)|^2} + E \left\{ |y|^2 |m|^2 |v(t)|^2 \right\} \\
 &\quad + E \left\{ n^*(t) y(t) m v(t) + n(t) y^*(t) m^* v^*(t) \right\}
 \end{aligned}
 \tag{33}$$

(S) The performance of the SLC can be readily analyzed when the response time of the loop is very slow compared to the inverse bandwidth of the waveforms  $n(t)$  and  $v(t)$ . From the differential equation of the control loop, (31), we see that the response time of the loop depends upon the fixed parameters  $T$  and  $A$  and also on the power level in the auxiliary channel  $|v|^2$ . By using a very narrow band integrating filter, "T" can be made large enough so that the loop response time is very slow for some power level,  $|v(t)|^2$ .

(S) With slow loop response the control voltage,  $x(t)$ , (and hence  $y(t)$ ), will not be correlated with either  $n(t)$  or  $v(t)$ . Hence equation (33) would then reduce to



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$$E \left\{ |r(t)|^2 \right\} = \overline{|n(t)|^2} + E \left\{ |y(t)|^2 \right\} |m|^2 \cdot \overline{|v(t)|^2} \quad (34)$$

(S) In this case we see that  $n(t)$ , the part of the main channel signal that is uncorrelated with the auxiliary channel waveform,  $v(t)$ , passes through the SLC unattenuated. The remainder,  $mv(t)$ , is reduced in power by  $E \left\{ |y(t)|^2 \right\}$ . The quantity,  $E \left\{ |y(t)|^2 \right\}$ , therefore, represents the cancellation ratio. Still assuming slow loop response let

$$\psi(t) = E \left\{ y(t) \right\}$$

and apply the expectation operator to both sides of (31). This gives

$$T \frac{d\psi}{dt} + (1 + A \overline{|v|^2}) \psi = 1 \quad (35)$$

(S) This is the differential equation for simple exponential behavior with time constant,

$$\tau = \frac{T}{1 + A \overline{|v|^2}} \quad (36)$$

(S) From this we note that the loop bandwidth is larger than the bandwidth of the integrating filter by the factor  $(1 + A \overline{|v|^2})$ . For convenience we refer to the loop bandwidth as the "enhanced" bandwidth,  $B_e$  and from (17) and (36), we have,

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$$B_e = (1 + A|v|^2) B_I \quad (37)$$

(S) The transient response from the initial condition  $\psi_0 = 1$  corresponding to  $y_0 = 1$  or  $x_0 = 0$ , is

$$\psi(t) = \frac{1 + A|v|^2 e^{-t/\tau}}{1 + A|v|^2} \quad (38)$$

The steady-state value is,

$$E_\infty(y) = \psi(\infty) = \frac{1}{1 + A|v|^2} \quad (39)$$

(S) With very slow loop response, (small  $B_e$ ), the fluctuation in  $y(t)$  about its expected value,  $\psi(t)$ , will be negligible so that we have approximately,

$$E \left\{ |y(t)|^2 \right\} \approx \left\{ E(y(t)) \right\}^2 = \psi^2(t)$$

(S) Hence from equations (34) and (38) we conclude that the correlated component of the residue  $m v(t)$  will behave transiently as  $e^{-t/\tau}$ . Thus the residue is reduced at the rate of  $20 \log_{10} e^{-\pi B_e}$ , or  $27.2 B_e$  db/second. The steady-state value of  $E \left\{ |y(t)|^2 \right\}$  will be approximately,

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$$E_{\infty} \left\{ |y|^2 \right\} = |\psi(\infty)|^2 = \left( \frac{1}{1 + A|v|^2} \right)^2 \quad (40)$$

(S) The steady-state residue from (34) will therefore be,

$$E_{\infty} \left\{ |r|^2 \right\} = \overline{|n(t)|^2} + \left( \frac{1}{1 + A|v|^2} \right)^2 |m|^2 \overline{|v(t)|^2} \quad (41)$$

(S) Using equations (26) - (28) we may express (41) in terms of the jamming and receiver noise power levels,

$$E_{\infty} \left\{ |r|^2 \right\} = N_m + \frac{|m|^2 P_J}{1 + P_J/N_a} + \frac{P_J/N_a}{1 + P_J/N_a} \cdot \frac{|m|^2 P_J}{[1 + A(P_J + N_a)]^2} \quad (42)$$

(S) Comparing (42) with (13) which gives the minimum residue achievable, we see that the jammer residue now depends upon the loop gain  $A(P_J + N_a)$  as well as the ratio of  $P_J$  to  $N_a$ . Equation (42) is graphed in Figure (4) under the assumption that  $|m| = 1$  and  $N_a = N_m = P_n$ . For an optimum ideal canceller, the residue never exceeds the receiver noise level by more than 3 db. From Figure (4) we see that if the "Quiescent" loop gain,  $AP_n$ , is 0 db or greater, the residue output of the SLC will not rise above 3 db for any jammer power level. However, when the jammer power level is 20 db above the noise, the loop gain will be 40 db, assuming 0 db quiescent gain. This is about as high as one could safely expect to go before stability problems will show up. This problem can be averted by using a limiter in the auxiliary arm just ahead of the correlator. With the limiter the loop power gain will increase directly as the jammer

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power, instead of as its square. Hence, with a limiter and 0 db quiescent gain, the residue will be approximately 3 db up to  $P_J/N_a = 40$  db. Beyond this level overload limiters ahead of the SLC may be required to maintain stability.

(S) It should be emphasized that the results discussed above, particularly equation (42) and Figure (4) are valid only when the loop response time is slow compared to the inverse bandwidth of the signal channel. In the remainder of this report, the SLC is analyzed without making the slow response assumption.

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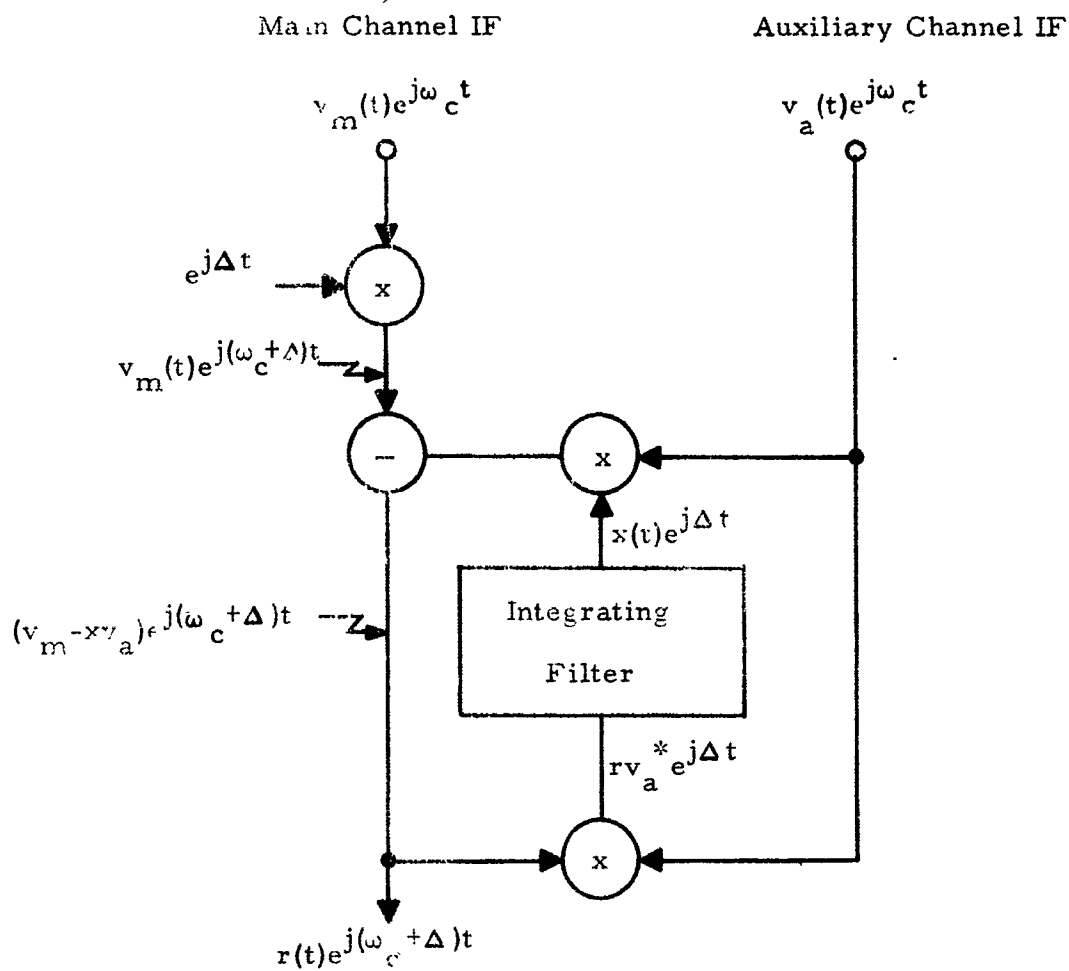


FIGURE 2 SIDELOBE CANCELLER BLOCK DIAGRAM

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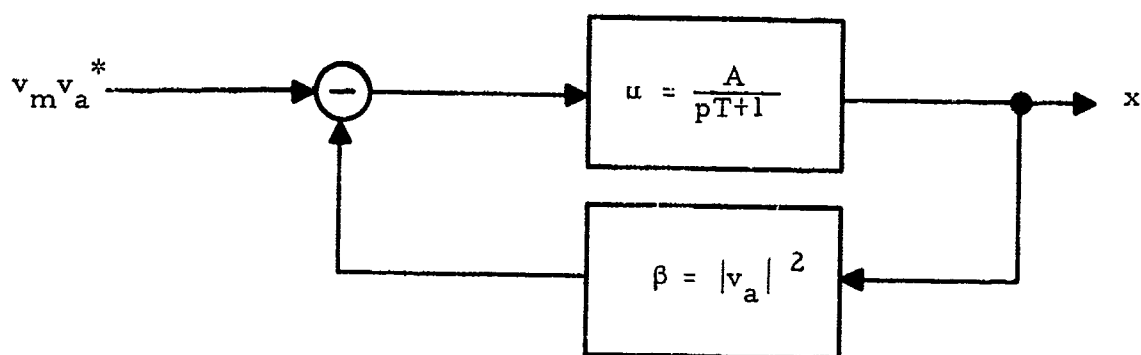


FIGURE 3 -- STANDARD "FEEDBACK" MODEL OF CANCELLER

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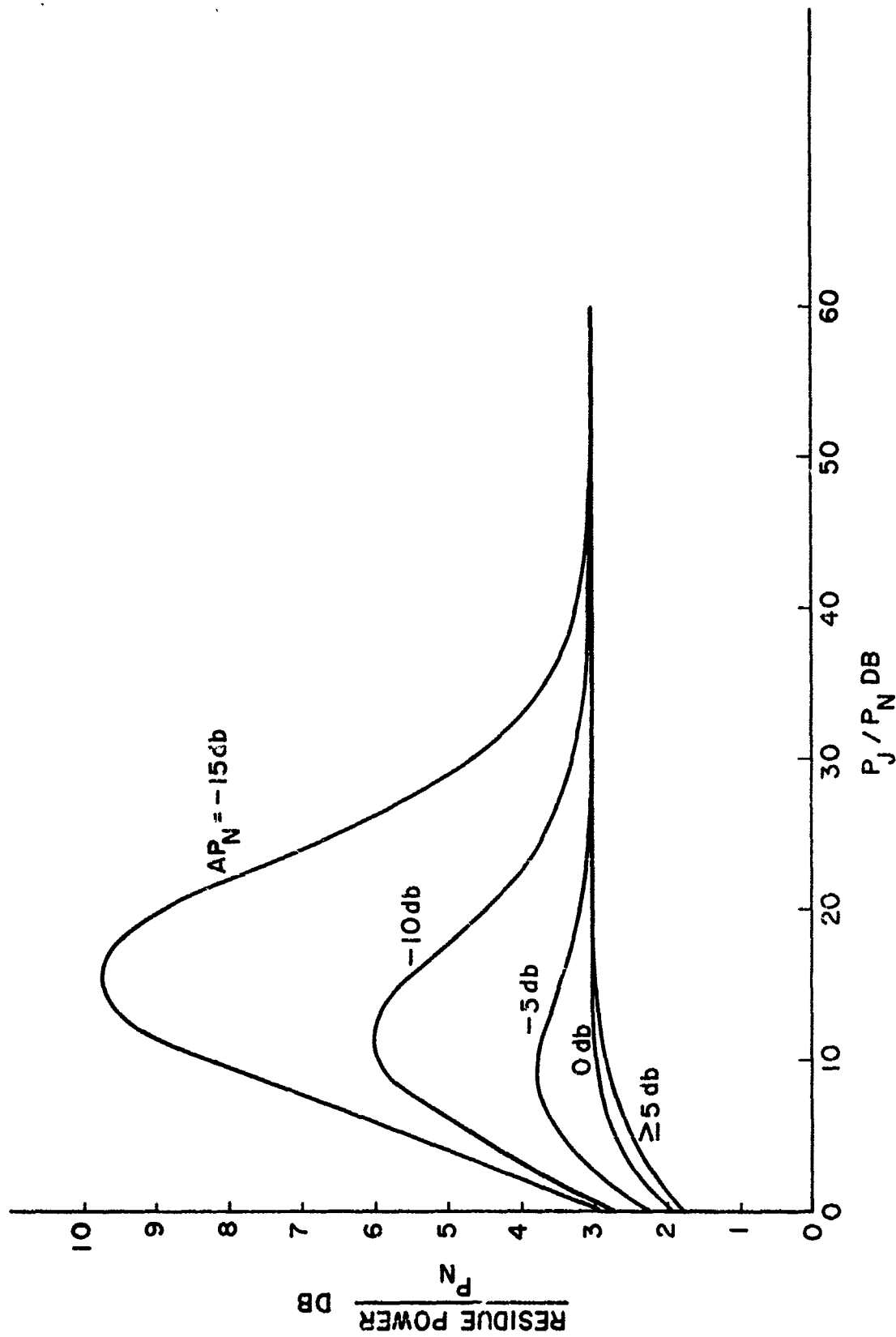


Figure 4. STEADY-STATE PERFORMANCE OF SLOW SLC

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### IV. Steady-State Performance

(S) Although the differential equation describing the loop behavior, equation (31), is linear and of first order; the presence of the random time-varying coefficient makes it very difficult to obtain exact solutions for the expected value of the output residue power. It is not too difficult however, to obtain useful upper and lower bounds on the residue.

(S) When the loop response time is very slow compared to the inverse bandwidth of the inputs to the SLC, the output residue consists of a linear combination of  $n(t)$  and  $v(t)$ , (using the canonical representation). This suggests that a lower bound to the residue output of the SLC can be obtained by finding the portion of the residue that is linearly correlated to  $n(t)$  or  $v(t)$ . That is, we represent the residue output as,

$$r(t) = \gamma_n(t) n(t) + m \gamma_v(t) v(t) + \xi(t) \quad (43)$$

$$\text{where } \gamma_n(t) = \frac{E(r(t) n^*(t))}{E(|n(t)|^2)} \quad (44)$$

$$\gamma_v(t) = \frac{E(r(t) m^* v^*(t))}{|m|^2 E(|v(t)|^2)} \quad (45)$$

and  $\xi(t)$  = a random waveform not correlated with either  $n(t)$  or  $v(t)$

The output residue power will be greater than the power of the first two terms in (43), thus,

$$E\{|r(t)|^2\} \geq |\gamma_n(t)|^2 E\{|n(t)|^2\} + |m|^2 |\gamma_v(t)|^2 E\{|v(t)|^2\} \quad (46)$$



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Now, using equation (32) for the output residue, equations (42) and (43) reduce to,

$$\gamma_n(t) = 1 + m \frac{E(y(t) v(t) n^*(t))}{E(|n(t)|^2)} \quad (47)$$

$$\text{and } \gamma_v(t) = \frac{E(y(t) |v(t)|^2)}{E(|v(t)|^2)} \quad (48)$$

(S) For the steady-state performance we are interested in the limits approached by  $\gamma_v$  and  $\gamma_n$  as  $t$  approaches infinity. To find the limit of  $\gamma_v$  we note first that since the SLC loop is stable,

$$\lim_{t \rightarrow \infty} E\left\{\frac{dy}{dt}\right\} = E_{\infty}\left\{\frac{dy}{dt}\right\} = 0 \quad (49)$$

(S) Now, applying the expectation operator to both sides of the loop differential equation, (31), letting 't' approach infinity and using (49), we get

$$\begin{aligned} E_{\infty}\left\{(1 + A|v(t)|^2) y(t)\right\} &= 1 - \frac{AE_{\infty}(n(t)v^*(t))}{m} \\ &= 1 \end{aligned} \quad (50)$$

since  $n(t)$  and  $v(t)$  are uncorrelated. From this we get,

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$$E_{\infty} \left\{ y(t) |v(t)|^2 \right\} = \frac{1 - E_{\infty}(y(t))}{A} \quad (51)$$

and therefore,

$$\Gamma_v = \lim_{t \rightarrow \infty} \gamma_v(t) = \frac{1 - E_{\infty}(y(t))}{A|v|^2} \quad (52)$$

(S) To evaluate  $E_{\infty}(y)$ , we first write the general solution to the loop differential equation (31) in integral form,

$$\begin{aligned} y(t) = & y(0) e^{-\int_0^t \frac{1 + A|v(\theta)|^2}{T} d\theta} + \\ & + \frac{1}{T} \int_0^t \left( \frac{1 - A n(u) v^*(u)}{m} \right) e^{-\int_0^t \frac{1 + A|v(\theta)|^2}{T} d\theta} du \end{aligned} \quad (53)$$

(S) If the initial value of the control voltage  $x$  is zero, the initial value of  $y$  will be 1, that is  $y(0) = 1$ .

(S) The first term on the r. h. s. of (53) disappears as  $t \rightarrow \infty$  so that applying the expectation operator to (53) and letting  $t \rightarrow \infty$ , we get,

$$E_{\infty} \left\{ y(t) \right\} = \lim_{t \rightarrow \infty} \frac{1}{T} \int_0^t E \left\{ e^{-\int_u^t \frac{1 + A|v(\theta)|^2}{T} d\theta} \right\} du \quad (54)$$

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(S) The integral in (54) is evaluated in Appendix (A) under the assumption that  $v(t)$  is a gaussian random process with an exponentially decreasing correlation function. The result is given by,

$$E_{\infty} \left\{ y(t) \right\} = \frac{1}{A|v|^2} \frac{Z}{p} F(1, p; p+1; Z^2) \quad (55)$$

where  $F( )$  = hypergeometric function

$$p = \frac{\beta - \alpha + 1}{2\beta}$$

$$Z = \frac{\beta - \alpha}{\beta + \alpha}$$

$$\alpha = \pi B_J T \quad (B_J = 3 \text{ db bandwidth of the waveforms } n(t) \text{ and } v(t))$$

$$\text{and, } \beta^2 = \alpha^2 + 2\alpha \overline{A|v|^2}$$

(S) The limit approached by  $y_n(t)$  as  $t$  approaches infinity can also be evaluated from (53). We first obtain,

$$\begin{aligned} E_{\infty} \left\{ y(t) v(t) n^*(t) \right\} &= \frac{A}{mT} \lim_{t \rightarrow \infty} \int_0^t E \left\{ n^*(t) n(u) v(t) v^*(u) e^{-\int_u^t \frac{1+A|v(\theta)|^2}{T} d\theta} \right\} du \\ &= - \frac{A}{mT} \overline{|n|^2} \lim_{t \rightarrow \infty} \int_u^t e^{-\alpha(t-u)} E \left\{ v(t) v^*(u) e^{-\int_0^t \frac{1+A|v(\theta)|^2}{T} d\theta} \right\} du \end{aligned} \quad (56)$$

(S) The integral on the r. h. s. of (56) is evaluated in Appendix (A).

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The result is,

$$E_{\infty} \left\{ y(t) v(t) n^*(t) \right\} = - \frac{|n|^2}{m} \frac{Z(1-Z^2)}{p} F(2, p; p+1; Z^2) \quad (57)$$

$$\text{where } p = \frac{1 + 2\beta}{2\beta}$$

and,  $\alpha, \beta, Z$  are defined above under (55)

(S) Using this result in (47), we obtain,

$$\Gamma_n = \lim_{t \rightarrow \infty} \gamma_n(t) = (1-Z) \left\{ 1 - \frac{Z(1+Z)(1-p)}{p} F(1, p; p+1; Z^2) \right\} \quad (58)$$

(S) This equation is graphed in Figure (5). Here  $\Gamma_n$  is plotted as a function of  $(1 + A |v|^2)$  which is  $1 + \text{loop gain}$ . Contours of constant  $\alpha (=B_J/B_I)^1$  and of constant ratio of  $B_e/B_J (= \frac{1+A|v|^2}{\alpha})$  are shown. For large loop gain  $\Gamma_n$  is essentially a function of the ratio  $B_e/B_J$ .  $\Gamma_n$  is always less than unity which indicates that the uncorrelated component of the main channel signal,  $n(t)$  may also be cancelled somewhat. The amount of cancellation is small if  $B_e/B_J$  is small compared with unity. When the parameters are such that  $n(t)$  is being cancelled we naturally find that desired signal is also cancelled.

(S) Figure (6) is a plot of  $(1 + A |v|^2) \Gamma_v$  as a function of  $(1 + A |v|^2)$ . This quantity is always less than 1 and as can be seen, for values of  $\alpha > 1$ , is very close to unity. Thus  $\Gamma_v$  is approximately the inverse of  $1 + A |v|^2$ .

---

<sup>1</sup> the ratio of the bandwidth of  $n(t)$  or  $v(t)$  to the integrating filter bandwidth.

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(S) To obtain an upper bound for the steady-state output residue power we start with the differential equation for the loop, (31) which we repeat here for convenience.

$$T \frac{dy}{dt} + (1 + A|v(t)|^2) y = 1 - \frac{An(t) v^*(t)}{m} \quad (31)$$

Now multiply (31) by  $y^*(t)$ ,

$$T y^* \frac{dy}{dt} + (1 + A|v(t)|^2) |y|^2 = y - \frac{An(t) v^*(t) y^*(t)}{m} \quad (59)$$

the conjugate of (59) is,

$$T y \frac{dy^*}{dt} + (1 + A|v(t)|^2) |y|^2 = y^* - \frac{An^*(t) v(t) y(t)}{m} \quad (60)$$

Adding (59) and (60) yields,

$$T \frac{d|y|^2}{dt} + 2(1 + A|v(t)|^2) |y|^2 = y + y^* - \frac{An(t) v^*(t) y^*(t)}{m} - \frac{An^*(t) v(t) y(t)}{m} \quad (61)$$

(S) Now apply the expectation operator to (61) and let  $t \rightarrow \infty$ . Noting that  $E\infty(y)$  is real (from (55)) and that,

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$$\lim_{t \rightarrow \infty} E \left\{ \frac{d|y|^2}{dt} \right\} = E_{\infty} \left\{ \frac{d|y|^2}{dt} \right\} = 0 \quad (62)$$

we obtain,

$$\begin{aligned} 2E_{\infty}(|y|^2) + 2A E_{\infty}(|y|^2|v|^2) &= 2E_{\infty}(y) - \frac{A}{m} E_{\infty}(y^* v^* n) \\ &- \frac{A}{m^*} E_{\infty}(y v n^*) \end{aligned} \quad (63)$$

(S) The expected steady-state output residue is given by equation (33) which we repeat below,

$$\begin{aligned} E_{\infty} \left\{ |r|^2 \right\} &= \overline{|n|^2} + m E_{\infty}(y v n^*) + m^* E_{\infty}(y^* v^* n) \\ &+ |m|^2 E_{\infty}(|y|^2|v|^2) \end{aligned} \quad (33)$$

(S) If we solve (63) for  $E_{\infty}(|y|^2|v|^2)$  and substitute into (33) we get,

$$\begin{aligned} E_{\infty} \left\{ |r|^2 \right\} &= \overline{|n|^2} + \frac{|m|^2 (E_{\infty}(y) - E_{\infty}|y|^2)}{A} \\ &+ \frac{m}{2} E_{\infty}(y v n^*) + \frac{m^*}{2} E_{\infty}(y^* v^* n) \end{aligned} \quad (64)$$

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(S) From equations (47) and (57) however we note that,

$$\Gamma_n = \lim_{t \rightarrow \infty} \gamma_n(t) = \frac{\overline{|n|^2} + m E_{\infty}(y v n^*)}{\overline{|n|^2}} \quad \text{is real,}$$

hence (64) becomes,

$$E_{\infty} \{ |r|^2 \} = \frac{|m|^2}{A} \frac{E_{\infty}(y) - E_{\infty}(|v|^2)}{A} + \Gamma_n \overline{|n|^2} \quad (65)$$

(S) Equation (65) is exact. The quantity that is difficult to evaluate is  $E_{\infty} \{ |y|^2 \}$ . To get an upper bound on  $E_{\infty} \{ |r|^2 \}$  we may use the inequality,

$$E_{\infty} \{ |y|^2 \} \geq |E_{\infty}(y)|^2 \quad (66)$$

Therefore,

$$E_{\infty} \{ |r|^2 \} \leq \Gamma_n \overline{|n|^2} + \frac{|m|^2}{A} \{ E_{\infty}(y) - (E_{\infty}(y))^2 \} \quad (67)$$

or using equation (52),

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$$E_{\infty} \left\{ |r|^2 \right\} \leq \Gamma_n |m|^2 + E_{\infty}(y) \Gamma_v |m|^2 \overline{|v|^2} \quad (68)$$

(S) Equation (68) gives the desired upper bound on the expected output residue power. From (46) and (68) we see that the cancellation ratio,  $C_n$ , for the uncorrelated component  $|n|^2$  falls between the bounds,

$$\Gamma_n^2 \leq C_n \leq \Gamma_n \quad (69)$$

whereas  $C_v$ , the cancellation ratio of the correlated component falls between the bounds,

$$\Gamma_v^2 \leq C_v \leq E_{\infty}(y) \Gamma_v \quad (70)$$

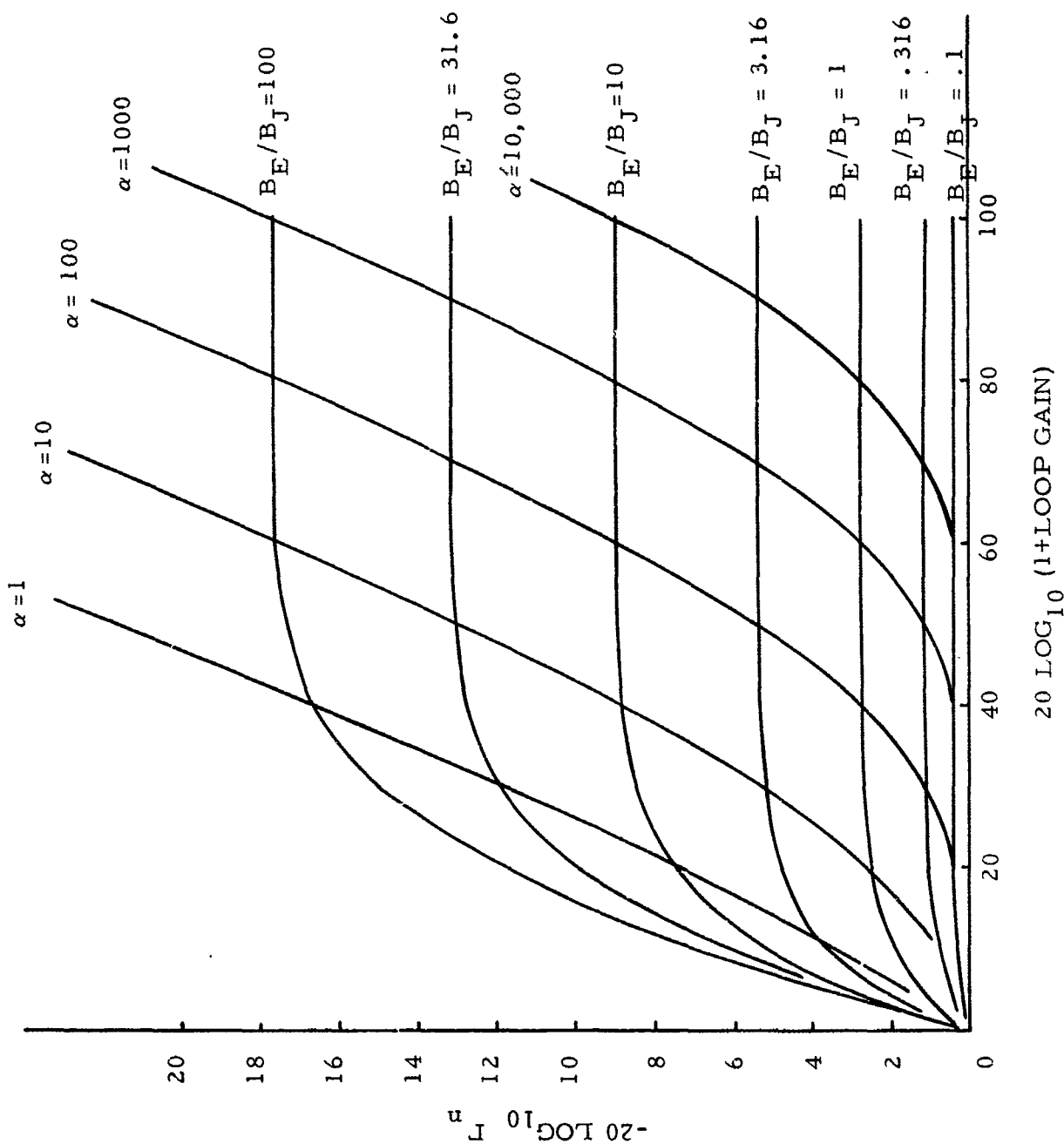
(S) The lower bounds on  $C_n$  and  $C_v$  are given in Figures (5) and (6) respectively. The upper bound on  $C_n$  can be obtained from Figure (5) by halving the ordinate scale. The upper bound on  $C_v$  may be obtained from Figure (7) which is a plot of  $(1 + A|v|^2) E_{\infty}(y) \Gamma_v$  vs.  $(1 + A|v|^2)$  for various values of the ratio of  $B_e/B_J$ . These values are always greater than unity indicating that the cancellation ratio,  $C_v$ , may be less than the inverse of  $(1 + A|v|^2)$ . However, if  $B_e/B_J$  is less than 1, the "loss" in cancellation is less than 1 db.



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FIGURE 5 --  $\Gamma_n$

CANCELLATION OF UNCORRELATED  
NOISE (LOWER BOUND)



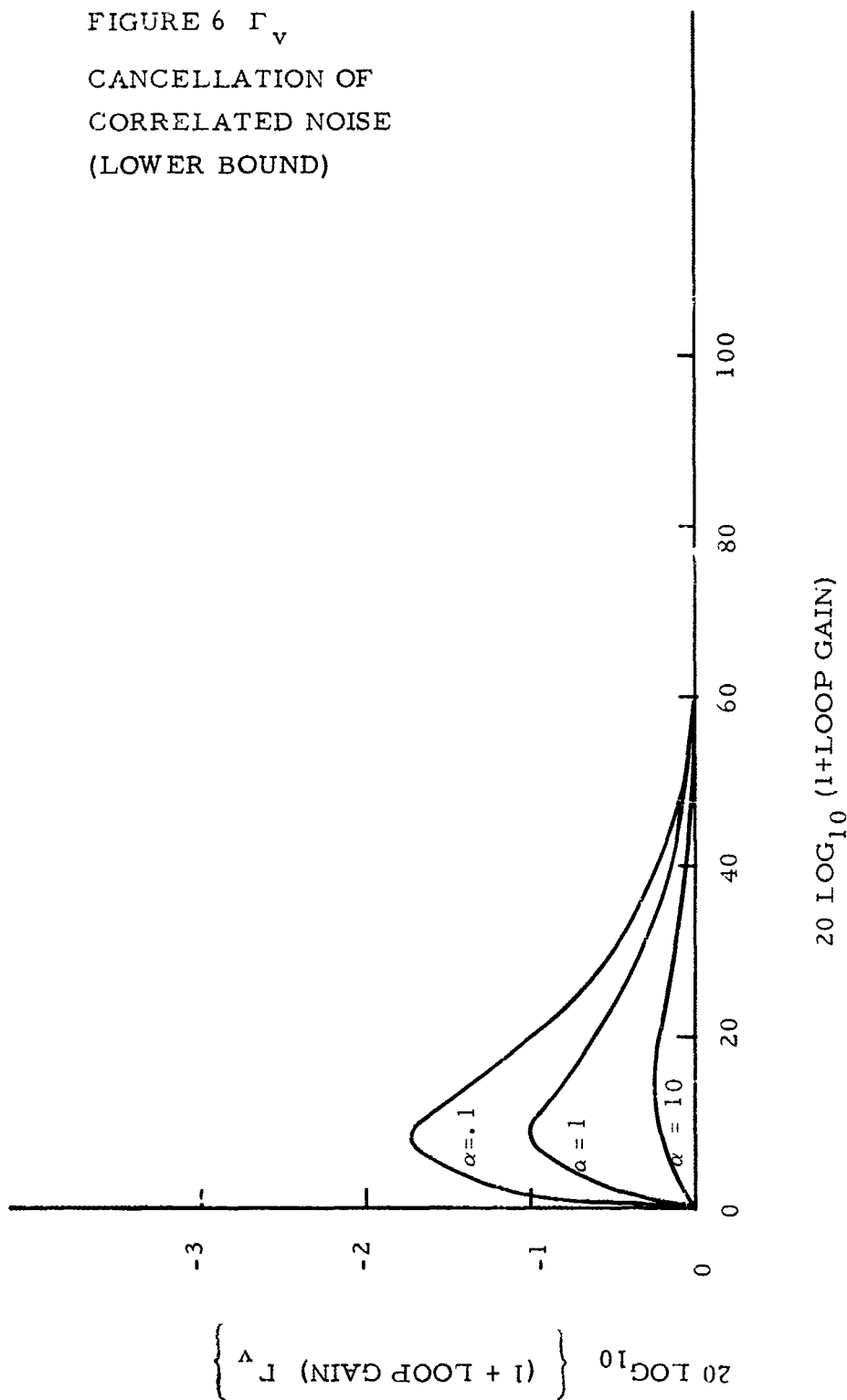
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FIGURE 6  $\Gamma_v$   
CANCELLATION OF  
CORRELATED NOISE  
(LOWER BOUND)



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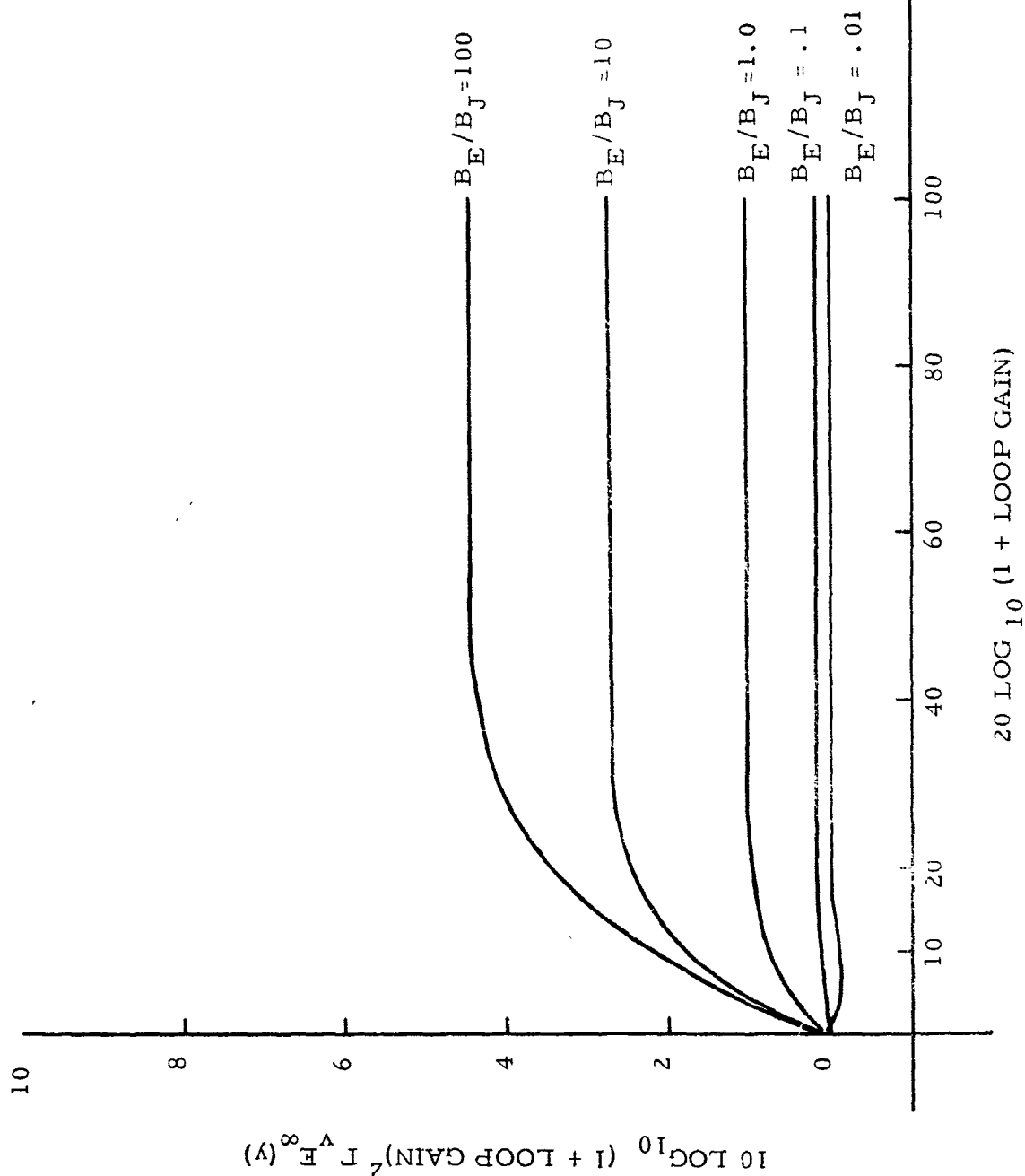
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FIGURE 7 --  $\Gamma_V E_\infty (v)$

CANCELLATION OF CORRELATED  
NOISE (UPPER BOUND)



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### V. Signal Cancellation

(S) In the previous section we saw that as the loop gain is increased the SLC causes some cancellation of the uncorrelated portion of the input. The amount of such cancellation increases with loop gain and also with the ratio of enhanced loop bandwidth to input bandwidth  $B_e/B_J$ . This aspect of the behavior of the SLC makes it necessary to investigate the effect of the SLC upon desired signal in the main lobe of the main antenna. We assume, as indicated in Figure V-a, that the desired signal is only present in the main channel. This will be essentially true in any practical SLC system since the gain of the auxiliary antenna will be about 20 db below the main lobe gain of the main antenna.

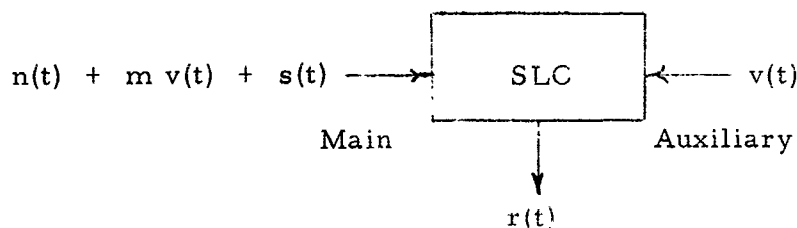


Figure V-a.

(S) The output of the SLC,  $r(t)$ , is given by

$$r(t) = n(t) + s(t) + y(t)m v(t) \quad (73)$$

(S) We assume that the signal arrives "long" after the loop has locked onto the jammer so that  $y(t)$  may be considered to be in its "steady state." Hence, from equation (53), replacing  $n(u)$  by  $n(u) + s(u)$  we obtain

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$$\text{for } t \rightarrow \infty, y(t) = \frac{1}{T} \int_0^t \left( 1 - \frac{A n(u) v^*(u)}{m} - \frac{A s(u) v^*(u)}{m} \right) e^{-\int_u^t \frac{1 + A|v|^2}{T} d\theta} du \quad (74)$$

The signal portion of  $r(t)$  is clearly (for  $t \rightarrow \infty$ ),

$$r_s(t) = s(t) + \frac{A}{T} \int_0^t s(u) v(t) v^*(u) e^{-\int_u^t \frac{1 + A|v|^2}{T} d\theta} du \quad (75)$$

(S) For detecting the presence of signal or estimating any of its parameters, the linear signal component in  $r_s(t)$  is of importance (that is  $r_s(t)$  rather than  $r_s^2(t)$ ). Since  $r_s(t)$  contains a stochastic parameter,  $v(t)$ , we find its expected value,

$$E \left\{ r_s(t) \right\} = s(t) - \frac{A}{T} \int_0^t s(u) E \left\{ v(t) v^*(u) e^{-\int_u^t \frac{1 + A|v|^2}{T} d\theta} \right\} du \quad (76)$$

From Appendix (A), we have

$$\frac{A}{T} E \left\{ v(t) v^*(u) e^{-\int_u^t \frac{1 + A|v|^2}{T} d\theta} \right\} = w(t-u) \quad (77)$$

$$\text{where } w(x) = \frac{A|v|^2}{T} \left\{ \frac{2 \alpha \beta e^{(\alpha-1) \frac{x}{T}}}{(\alpha^2 + \beta^2) \sinh \frac{\beta x}{T} + 2 \alpha \beta \cosh \frac{\beta x}{T}} \right\}^2 \quad (78)$$

$$\alpha = \pi \beta_J T$$

$$\text{and, } \beta^2 = \alpha^2 + 2 \alpha A|v|^2$$

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(S) We can therefore write equation (76) as,

$$E \{ r_s(t) \} = \int_0^t s(u) h(t-u) du \quad (79)$$

$$\text{where } h(t) = \delta(t) - w(t) \quad (80)$$

(S) Thus from (79) we see that the SLC behaves like a linear filter acting on the signal. The filter is specified by its impulse response  $h(t)$ . The transfer function is given simply by,

$$H(f) = \int_0^{\infty} h(t) e^{-j2\pi ft} dt \quad (81)$$

(S) Investigation of  $H(f)$ , (see Appendix (B) ), shows that its bandwidth is very wide, approximating the bandwidth of the input noise (jamming and receiver). Assuming that the signal bandwidth is narrow compared to the jammer bandwidth and that the signal bandcenter is approximately at the center of the jammer band, the signal cancellation will simply be  $H(0)$ , where from (81),

$$H(0) = \int_0^{\infty} h(t) dt \quad (82)$$

or using (80),

$$H(0) = 1 - \int_0^{\infty} w(t) dt \quad (83)$$

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(S) The integral in (83) can be expressed in terms of a hypergeometric function as,

$$\int_0^{\infty} w(t) dt = \frac{Z(1-Z^2)}{p} F(2, p, p+1, Z^2) \quad (84)$$

$$\text{where } Z = \frac{\beta - \alpha}{\beta + \alpha}$$

$$\text{and } p = \frac{2\beta - \alpha + 1}{2\beta} \quad (85)$$

(S) The signal cancellation factor,  $H(o)$ , is shown in Figure (8), plotted against the noise cancellation ratio,  $\Gamma_v$ . Two sets of curves are shown; on one set the parameter is  $\alpha$  ( $=\pi B_J T$ ), on the other the bandwidth ratio ( $B_e/B_J$ ) is the parameter. From the Figure we see that the bandwidth ratio should be kept under unity to prevent signal cancellation from exceeding 4 db.

(S) Figure (9) summarizes the important results obtained thus far. It shows the variation of signal cancellation, with the bandwidth ratio  $B_e/B_J$ , upper and lower bounds on cancellation of the uncorrelated noise component, and the maximum "loss" in cancellation of the correlated components. The results shown in this Figure are the asymptotic values for large loop gain.

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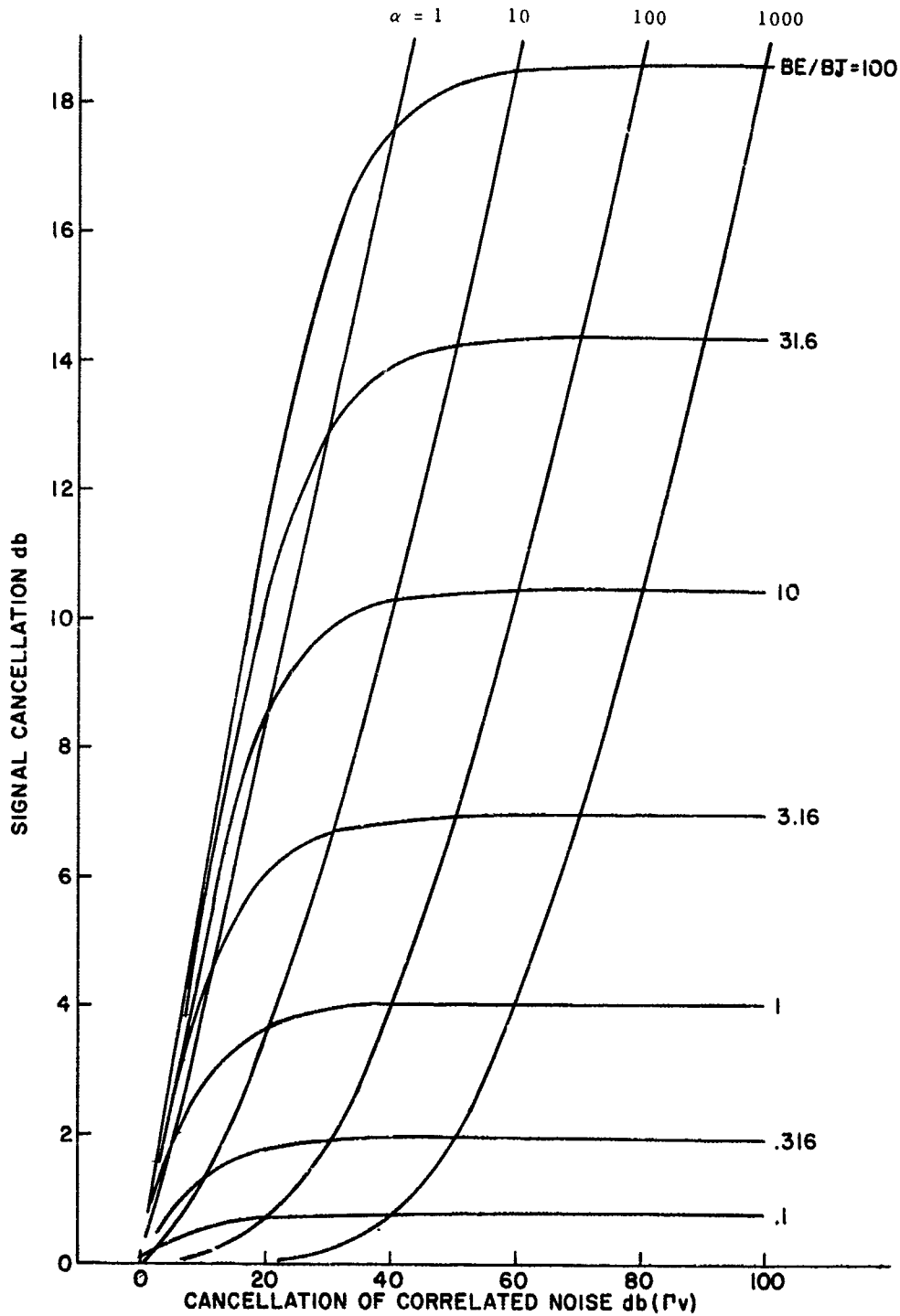


Figure 8. SIGNAL CANCELLATION

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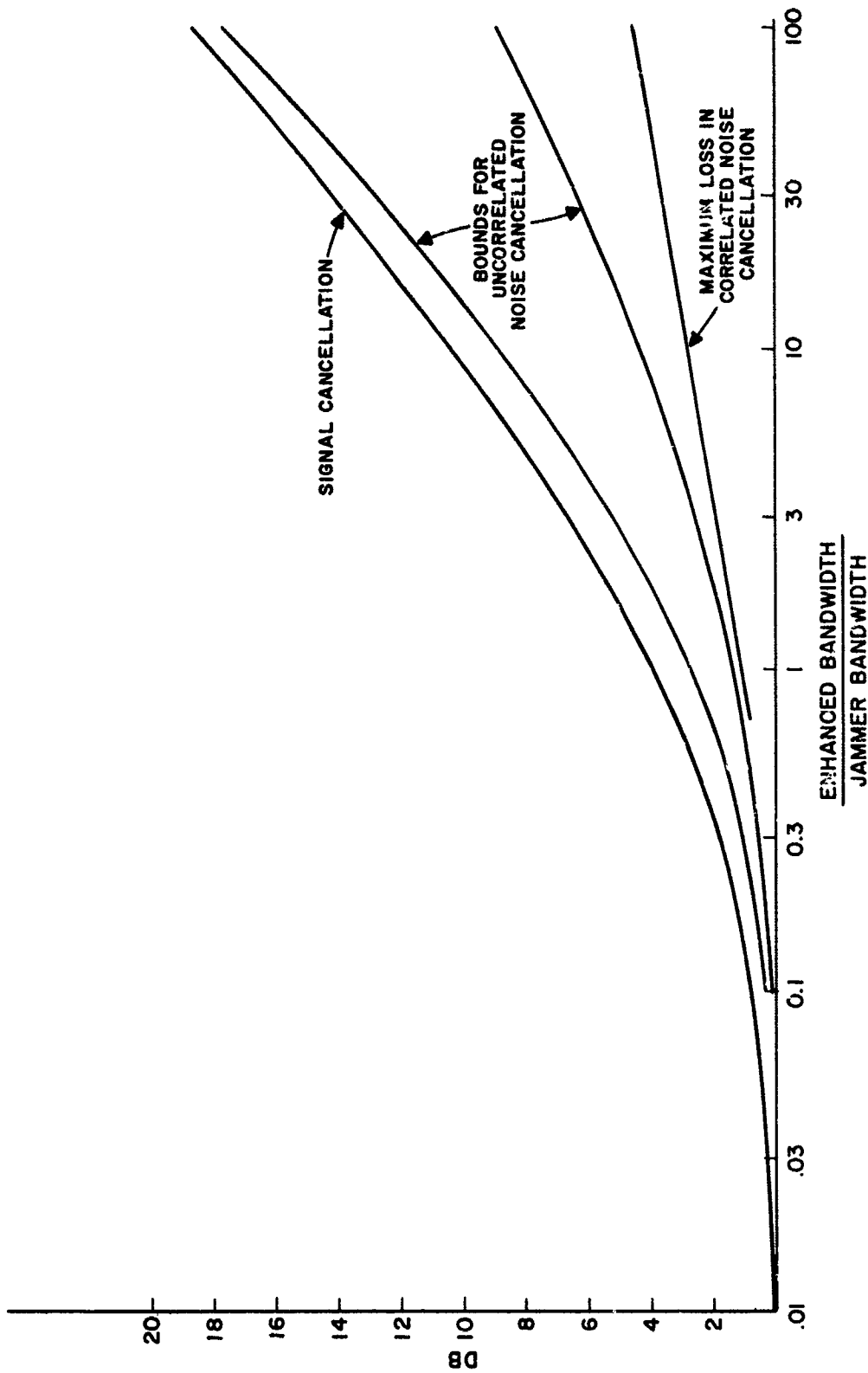


Figure 9. HIGH GAIN PERFORMANCE OF SLC

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## VI. Transient Response

(S) In this section we obtain the transient response of the SLC. The transient response that is easiest to measure in the laboratory is that of the control voltage,  $y(t)$ , (actually  $x(t)$  in the real world). The quantity of real interest is the time response of the output residue power, but like the steady-state response we can only obtain an estimate (much better in this case than in the steady-state case). A closely related quantity that we can evaluate exactly is the transient response of  $\gamma_v(t)$ , which is a normalized measure of the linear jamming component in the output residue.

(S) The expected value of the normalized control voltage can be obtained directly from (11),

$$E \{ y(t) \} = E \left\{ e^{-\int_0^t \frac{1 + A|v(\theta)|^2}{T} d\theta} \right\} \quad (86)$$

$$+ E \left\{ \frac{1}{T} \int_0^t e^{-\int_u^t \frac{1 + A|v(\theta)|^2}{T} d\theta} du \right\}$$

$$\text{since, } E \{ n(u) v^*(u) \} = 0$$

(S) This can be readily evaluated using the results of Appendix A to give,

$$E \{ y(t) \} = E_{\infty}(y) + (1 - Z^2) e^{-2\beta p T} \left\{ \frac{1}{1 - Z^2 e^{-2\beta p T}} - \frac{1}{2\beta} F(1, p; p+1, Z^2 e^{-2\beta p T}) \right\} \quad (87)$$

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where  $\alpha = \pi B_J T$

$$\beta^2 = \alpha^2 + 2\alpha A \overline{|v|^2}$$

$$Z = \frac{\beta - \alpha}{\beta + \alpha}$$

$$p = \frac{1 + \beta - \alpha}{2\beta}$$

$$\tau = t/T$$

and  $E\omega(y) = E\{y(\infty)\}$  is given by equation (55).

(S) The transient response of  $\gamma_v(t)$ , which is defined as

$$\gamma_v(t) = \frac{E\{y(t)|v', |^2\}}{\overline{|v|^2}} \quad (88)$$

can be conveniently obtained from (87) above and the differential equation of the SLC, (31), which we may write as,

$$T e^{-t/T} \frac{d}{dt} \left\{ e^{t/T} y(t) \right\} + \frac{1}{\overline{|v|^2}} y(t) = 1 - A n(t) v^*(t) \quad (89)$$

(S) Applying the expectation operator to both sides of (89) and using the definition of  $\gamma_v(t)$ , we get

$$A \overline{|v|^2} \gamma_v(t) = 1 - T e^{-t/T} \frac{d}{dt} (e^{t/T} E\{y(t)\}) \quad (90)$$

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After differentiation and some manipulation, this yields,

$$y_v(t) = (1+Z)(1-Z^2) e^{-2\beta p \tau} \frac{1+Ze^{-2\beta \tau}}{(1-Z^2 e^{-2\beta \tau})^2} + \frac{1-E\{y(t)\}}{A|v|^2} \quad (91)$$

(S) The expected output residue power is given by equation (33) as,

$$\begin{aligned} E\{|r(t)|^2\} &= |n(t)|^2 + 2\text{Re } m E\{n^*(t)y(t)v(t)\} \\ &\quad + |m|^2 E\{|y(t)|^2 |v(t)|^2\} \end{aligned} \quad (92)$$

(S) The first two terms on the rhs of (92) are proportional to the uncorrelated component of the input power  $|n(t)|^2$ . Since our main interest is in the transient response to the jamming or correlated component  $|v(t)|^2$ , we shall simplify matters by assuming  $n(t)$  is zero. The output residue is then simply,

$$E\{|r_v(t)|^2\} = |m|^2 E\{|y_v(t)|^2 |v(t)|^2\} \quad (93)$$

where,

$$y_v(t) = e^{-\int_0^t \frac{1+A|v(\theta)|^2}{T} d\theta} + \frac{1}{T} \int_u^t \frac{1+A|v(\ell)|^2}{T} d\ell \quad du \quad (94)$$

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(S) The first term in (94) starts at unity and decays to zero. This represents the bulk of the transient. The second term starts at zero and builds up to the steady-state value which for all cases of interest is very small compared to unity. From equation (68), we note that

$$E\infty\left\{|r_v|^2\right\} = E\infty(y) \Gamma_v |m|^2 \overline{|v|^2} \quad (95)$$

(S) Hence we can approximate (93) by,

$$\begin{aligned} E\left\{|r_v(t)|^2\right\} &= |m|^2 E\left\{|v(t)|^2 e^{-2\int_0^t \frac{1+A|v(\theta)|^2}{T} d\theta}\right\} \\ &+ |m|^2 \overline{|v|^2} E_s\left\{y(t)\right\} \gamma_{vs}(t) \end{aligned} \quad (96)$$

where  $E_s\left\{y(t)\right\}$  = to the component of  $E\left\{y(t)\right\}$  that starts at zero and builds up to the steady-state value  $E\infty\left\{y\right\}$ .

and  $\gamma_{vs}(t)$  = component of  $\gamma_v(t)$  that starts at zero and builds up to the steady-state value  $\Gamma_v$ .

(S) From equations(87) and (91), we have,

$$E_s\left\{y(t)\right\} = E\infty(y) - \frac{(1-Z^2)e^{-2\beta p\tau}}{2\beta} F(1, p; p+1; Z^2 e^{-2\beta p\tau}) \quad (97)$$

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and,

$$\gamma_{vs}(t) = \frac{1 - E\{y(t)\}}{A|v|^2} \quad (98)$$

(S) Using the results of Appendix A we easily find,

$$E \left\{ |v(t)|^2 e^{-2 \int_0^t \frac{1+A|v(\tau)|^2}{T} d\tau} \right\} \quad (99)$$

$$= |v|^2 (1+Z) (1-Z)^2 e^{-(u-\alpha+2)\tau} \frac{1+Z e^{-2u\tau}}{(1-Z^2 e^{-2u\tau})^2}$$

$$\text{where } Z = \frac{u-\alpha}{u+\alpha} \quad (100)$$

$$\text{and } u = \alpha^2 + 4\alpha A \overline{|v|^2}$$

(S) The transient response  $E |r_v(t)|^2$  normalized by  $|m|^2 \overline{|v|^2}$  is plotted in Figure (10) against  $\pi B_E t$ . Curves are given for different values of the bandwidth ratio  $B_e/B_J$  assuming large loop gain. For  $B_e/B_J = 1$ , curves are shown for different values of the loop gain  $A|v|^2$ . As the enhanced bandwidth,  $B_e$ , is increased, the transient response speeds up but the improvement comes at a decreasing rate after the ratio of  $B_e/B_J$  exceeds unity. If the ratio of  $B_e/B_J$  is much less than unity, the transient response is exponential and lock-on occurs at the rate of  $27.2 B_e$  db. per second. That is the correlated portion of the main channel signal is reduced at that rate. The asymptotic lock-on rate for various ratios of  $B_e/B_J$  is given in Table 1 below,

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TABLE 1

$B_e/B_J$	$1/B_e$ Lock-on rate, db/sec.
$\ll 1$	27.4
1.0	15.9
10.0	7.4
100.0	2.7

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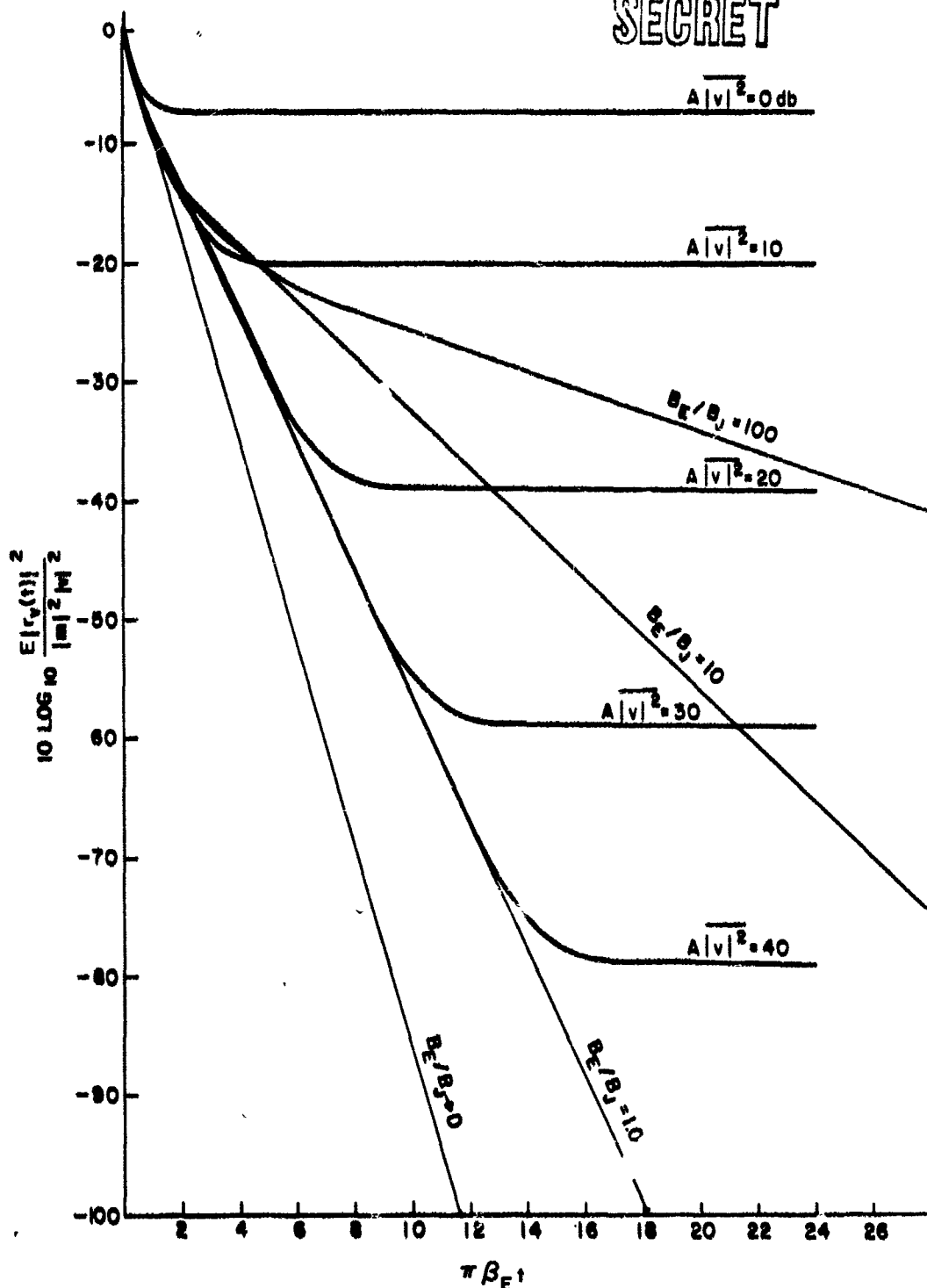


Figure 10. TRANSIENT RESPONSE

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## Appendix A Evaluation of Expectations

(U) In the main body of this report the evaluation of the expectation of a number of different expressions was required. All of these can be obtained from one basic form, namely,

$$L(A, B, a, b) = E \left\{ \exp \left[ Av(a) + Bv^*(b) - \int_a^b |v(\theta)|^2 d\theta \right] \right\} \quad A-1$$

where A, B are constants and a, b lie in the closed interval (u, t).

(U) From the basic form above we can easily obtain,

$$E \left\{ \exp - \int_a^b |v(\theta)|^2 d\theta \right\} = L(0, 0, a, b) \quad A-2$$

$$E \left\{ v(u) v^*(t) \exp - \int_a^b |v(\theta)|^2 d\theta \right\} = \frac{\partial^2 L}{\partial A \partial B} \bigg|_{0, 0, u, t} \quad A-3$$

or,

$$E \left\{ |v(t)|^2 \exp - \int_a^b |v(\theta)|^2 d\theta \right\} = \frac{\partial^2 L}{\partial A \partial B} \bigg|_{0, 0, t, t} \quad A-4$$

(U) To evaluate L we begin by expanding  $v(\theta)$  in a series of eigenfunctions of the integral,

$$\int_a^b R(\tau, \theta) \phi_k(\theta) d\theta = \lambda_k \phi_k(\tau) \quad A-5$$

$$\text{where } R(\tau, \theta) = E \left\{ v(\tau) v^*(\theta) \right\} \quad A-6$$

and the eigenfunctions,  $\phi_k$ , are normalized so that,

$$\int_u^t \phi_k(\theta) \phi_l^*(\theta) d\theta = \delta_{kl} \quad A-7$$

then,

$$v(\theta) = \sum_k Z_k \phi_k(\theta) \quad A-8$$

$$\text{where } Z_k = \int_u^t v(\theta) \phi_k^*(\theta) d\theta \quad A-9$$

(U) The coefficients,  $Z_k$ , are uncorrelated as is readily seen from,

$$\begin{aligned} E \left\{ Z_k Z_l^* \right\} &= \int_u^t \int_u^t E \left\{ v(\tau) v^*(\theta) \right\} \phi_k^*(\tau) \phi_l(\theta) d\theta d\tau \\ &= \int_u^t \int_u^t R(\tau, \theta) \phi_k^*(\tau) \phi_l(\theta) d\theta d\tau \quad A-10 \\ &= \lambda_l \int_u^t \phi_k^*(\tau) \theta_l(\tau) d\tau \\ &= \lambda_l \delta_{kl} \end{aligned}$$

(U) The distribution density of the  $Z_k$  may be shown to be multivariate Gaussian<sup>(3)</sup>. Therefore since the  $Z_k$  are uncorrelated they are independent.

(U) Since  $R(\tau, \theta)$ , ( $= R^*(\theta, \tau)$ ), is Hermitian the eigenvalues,  $\lambda_l$ , are all real.  $R(\tau, \theta)$  can be expressed as,

$$\begin{aligned} R(\tau, \theta) &= E \left\{ v(\tau) v^*(\theta) \right\} \\ &= \sum_k \lambda_k \phi_k(\tau) \phi_k^*(\theta) \end{aligned} \quad \text{A-11}$$

(U) The integral in the expression for L can be written as

$$\begin{aligned} \int_a^t |v(\theta)|^2 d\theta &= \int_a^t \sum_k \sum_l Z_k Z_l^* \phi_k(\theta) \phi_l^*(\theta) d\theta \\ &= \sum_k |Z_k|^2 \end{aligned} \quad \text{A-12}$$

and since,

$$\begin{aligned} v(a) &= \sum_k Z_k \phi_k(a) \\ v(b) &= \sum_k Z_k \phi_k(b) \end{aligned} \quad \text{A-13}$$

we may write equation A-1 as,

---

<sup>(3)</sup>Middleton, "Introduction to Statistical Communication Theory," p. 386

$$L = E \left\{ \exp - \sum_k \left[ |Z_k|^2 - AZ_k \phi_k(a) - BZ_k^* \phi_k^*(b) \right] \right\} \quad A-14$$

or since the  $Z_k$  are independent,

$$L = \prod_k E \left\{ \exp - \left[ |Z_k|^2 - AZ_k \phi_k(a) - BZ_k^* \phi_k^*(b) \right] \right\} \quad A-15$$

(U) Now each  $Z_k$  is a two-dimensional complex gaussian random variable, that is,

$$Z_k = x_k + i y_k \quad A-16$$

with probability density distribution,

$$\begin{aligned} p(x_k, y_k) &= \frac{1}{\pi \lambda_k} \exp - \frac{x_k^2 + y_k^2}{\lambda_k} \\ &= \frac{1}{\pi \lambda_k} \exp - \frac{|z_k|^2}{\lambda_k} \end{aligned} \quad A-17$$

(U) Hence we may write A-15 as,

$$L = \prod_k \frac{1}{\pi \lambda_k} \iint \exp - \left\{ \frac{1 + \lambda_k}{\lambda_k} |Z_k|^2 - AZ_k \phi_k(a) - BZ_k^* \phi_k^*(b) \right\} dx_k dy_k \quad A-18$$

$$= \prod_k \frac{1}{1 + \lambda_k} \exp \frac{\lambda_k}{1 + \lambda_k} A B \phi_k(a) \phi_k^*(b)$$

$$= \frac{\exp AB \sum_k \phi_k(a) \phi_k^*(b) x_k / (1 + \lambda_k)}{\prod_k (1 + \lambda_k)} \quad \text{A-18}$$

(U) To evaluate A-18 we may use well-known techniques.<sup>(4)</sup>  
First, we put,

$$\ln \prod_k (1 + \lambda_k) = \sum_k \ln(1 + \lambda_k) = \int_0^1 \sum_k \frac{\lambda_k}{1 + \lambda_k \xi} d\xi \quad \text{A-19}$$

and let,

$$G(\theta, \tau, \xi) = \sum_k \frac{\lambda_k}{1 + \lambda_k \xi} \phi_k(\theta) \phi_k^*(\tau) \quad \text{A-20}$$

Now consider the integral,

$$\begin{aligned} \int_u^t G(\theta, \tau, \xi) R(s, \theta) d\theta &= \xi \int_u^t \sum_k \frac{\lambda_k}{1 + \lambda_k \xi} \phi_k(\theta) \phi_k^*(\tau) \sum_l \lambda_l \phi_l(s) \phi_l^*(\theta) d\theta \\ &= \sum_k \frac{\xi \lambda_k^2}{1 + \lambda_k \xi} \phi_k(s) \phi_k^*(\tau) \end{aligned}$$

<sup>(4)</sup> Helstrom, "Statistical Theory of Signal Detection," Chap. 11.

$$= \sum_k \left( \lambda_k - \frac{\lambda_k}{1 + \lambda_k \xi} \right) \phi_k(s) \phi_k^*(\tau)$$

$$= R(s, \tau) - G(s, \tau, \xi) \quad A-21$$

(U) Hence the function  $G(\theta, \tau, v)$  defined by A-20 and required for the evaluation of A-18 may be obtained from the integral equation

$$G(s, \tau, \xi) + \xi \int_u^t G(\theta, \tau, \xi) R(s, \theta) d\theta = R(s, \tau) \quad A-22$$

(U) To solve this equation we must specify the correlation function,  $R(s, \tau)$ , which we take to be,

$$R(s, \tau) = \overline{|v|^2} e^{-\alpha |s - \tau|} \quad (5) \quad A-23$$

(U) This corresponds to a power spectrum of the form  $\frac{\overline{|v|^2}}{\alpha^2 + \omega^2}$ .

This is the shape of a single pole filter with a two-sided bandwidth of  $\frac{\alpha}{\pi}$ . The solution to A-22 is,

$$G(s, \tau, \xi) = \frac{2\alpha \overline{|v|^2}}{\beta} \frac{[\alpha \sinh \beta(s-u) + \beta \cosh \beta(s-u)] [\alpha \sinh \beta(t-\tau) + \beta \cosh \beta(t-\tau)]}{(\beta^2 + \alpha^2) \sinh \beta(t-u) + 2\alpha \beta \cosh \beta(t-u)}$$

A-24

(5) Note: Except where indicated otherwise  $\alpha$  and  $\beta$  in this appendix differ by a factor of  $T$  from the usage elsewhere in this report.

where,

$$u < s < T < t$$

and,

$$\beta^2 = \alpha^2 + 2\alpha \xi \overline{|v|^2}$$

(U) From A-20, A-24 we now obtain,

$$\begin{aligned} \sum_k \frac{\lambda_k}{1+\lambda_k \xi} &= \int_u^t G(s, s, \xi) ds \\ &= \frac{2\alpha \overline{|v|^2}}{\beta \left\{ (\beta^2 + \alpha^2) \sinh \beta(t-u) + 2\alpha \beta \cosh \beta(t-u) \right\}} \cdot \\ &\quad \cdot \int_u^t \left\{ \alpha \sinh \beta(s-u) + \beta \cosh \beta(s-u) \right\} \left\{ \alpha \sinh \beta(t-s) + \beta \cosh \beta(t-s) \right\} ds \end{aligned} \quad \text{A-25}$$

(U) Evaluating the integral in A-25 and then integrating the result with respect to  $\xi$  from 0 to 1 as indicated in A-19, we obtain

$$\prod_{k=1}^{\infty} (1+\lambda_k) = \frac{(\beta^2 + \alpha^2) \sinh \beta(t-u) + 2\alpha \beta \cosh \beta(t-u)}{2\alpha \beta e^{\alpha(t-u)}} \quad \text{A-26}$$

where now

$$\beta^2 = \alpha^2 + 2\alpha \overline{|v|^2}$$

(U) Now using A-2, A-18, and A-26, we obtain

$$E \left\{ \exp - \int_u^t |v(\theta)|^2 d\theta \right\} = \frac{1}{\prod_k (1 + \lambda_k)}$$

A-27

$$= \frac{2\alpha\beta e^{\alpha(t-u)}}{(\beta^2 + \alpha^2) \sinh\beta(t-u) + 2\alpha\beta \cosh\beta(t-u)}$$

Starting with A-3, we get,

$$E \left\{ v(a)v^*(b) \exp - \int_u^t |v(\theta)|^2 d\theta \right\} = \frac{\sum_k \phi_k(a)\phi_k^*(b)\lambda_k/(1 + \lambda_k)}{\prod_k (1 + \lambda_k)}$$

$$= \frac{G(a, b, 1)}{\prod_k (1 + \lambda_k)}$$

A-28

$$\frac{4\alpha^2 \overline{|v|}^2 e^{\alpha(t-u)} [\alpha \sinh\beta(a-u) + \beta \cosh\beta(a-u)] [\alpha \sinh\beta(t-b) + \beta \cosh\beta(t-b)]}{[(\alpha^2 + \beta^2) \sinh\beta(t-u) + 2\alpha\beta \cosh\beta(t-u)]^2}$$

(U) If we now set  $a=u$ ,  $b=t$  in A-28, we obtain

A-29

$$E \left\{ v(u)v^*(t) \exp - \int_u^t |v(\theta)|^2 d\theta \right\} = \frac{(2\alpha\beta)^2 \overline{|v|}^2 e^{\alpha(t-u)}}{[(\alpha^2 + \beta^2) \sinh\beta(t-u) + 2\alpha\beta \cosh\beta(t-u)]^2}$$



and setting  $a = b = t$ ,

$$E \left\{ |v(t)|^2 \exp - \int_u^t |v(\theta)|^2 d\theta \right\} = \frac{4\alpha^2 \beta |v|^2 e^{\alpha(t-u)} \{ \alpha \sinh \beta(t-u) + \beta \cosh \beta(t-u) \}}{[(\alpha^2 + \beta^2) \sinh \beta(t-u) + 2\alpha\beta \cosh \beta(t-u)]^2}$$

A-30

(U) In addition to the expectations obtained above, we also need their integrals. These can be determined by first expressing the expectations in terms of exponentials, e. g.

$$(\alpha^2 + \beta^2) \sinh \beta(x) + 2\alpha\beta \cosh \beta(x) = (\beta + \alpha)^2 \frac{e^{\beta x}}{2} \left( 1 - \left( \frac{\beta - \alpha}{\beta + \alpha} \right)^2 e^{-2\beta x} \right)$$

A-31

and then using the integration formula,

$$\int_0^t \frac{e^{-pt}}{(1 - Ze^{-t})^n} dt = \frac{1}{p} F(m, p; p+1; Z) -$$

A-32

$$- \frac{e^{-pt}}{p} F(n, p; p+1; Ze^{-pt})$$

where  $F(\ )$  is the Gaussian Hypergeometric function.

For example,

$$\frac{1}{T} \int_0^t e^{-\frac{t-u}{T}} E \left\{ \exp - \int_u^t |v(\theta)|^2 d\theta \right\} du =$$

A-9

$$\begin{aligned}
&= \frac{1}{T} \int_0^t \frac{2\alpha\beta e^{\alpha(t-u)} e^{-\frac{t-u}{T}} du}{(\alpha^2 + \beta^2) \sinh\beta(t-u) + 2\alpha\beta \cosh\beta(t-u)} \\
&= \frac{1}{T} \int_0^t \frac{2\alpha\beta e^{\alpha x} e^{-\frac{x}{T}} dx}{\frac{1}{2}(\beta + \alpha)^2 e^{\beta x} \left[1 - \left(\frac{\beta - \alpha}{\beta + \alpha}\right)^2 e^{-2\beta x}\right]} \\
&= \frac{1}{T} \frac{4\alpha\beta}{(\beta + \alpha)^2} \int_0^t \frac{e^{-(\beta - \alpha + \frac{1}{T})x}}{1 - \left(\frac{\beta - \alpha}{\beta + \alpha}\right)^2 e^{-2\beta x}} dx
\end{aligned}$$

(U) Letting  $2\beta x = y$  and then writing  $\alpha$  for  $\alpha T$ ,  $\beta$  for  $\beta T$ , and

$Z$  for  $\left(\frac{\beta - \alpha}{\beta + \alpha}\right)$ , A-34 becomes

$$= \frac{2\alpha}{(\beta + \alpha)^2} \int_0^{2\beta t} \frac{e^{-\left(\frac{\beta - \alpha + 1}{2\beta}\right)y}}{(1 - Ze^{-y})} dy \quad \text{A-35}$$

This is now in the form of equation A-32, with  $\alpha$  and  $\beta$  defined as they are in the main section of this report.

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## Appendix B

### "Signal Cancellation Filter"

(S) In the discussion on signal cancellation in section (V), we found that the SLC behaved like a filter with impulse response

$$h(t) = \delta(t) - w(t) \quad \text{B-1}$$

$$\text{where } w(t) = \frac{\overline{A|v|^2}}{T} \frac{(2\alpha\beta)^2 e^{(\alpha-1)t/T}}{[(\alpha^2 + \beta^2) \sinh \frac{\beta t}{T} + 2\alpha\beta \cosh \frac{\beta t}{T}]^2} \quad \text{B-2}$$

(U) In this appendix we wish to obtain at least a qualitative idea of the bandwidth of  $h(t)$  which is in effect the bandwidth of  $w(t)$ . The Laplace Transform of  $w(t)$  is

$$\begin{aligned} W(s) &= \int_0^{\infty} w(t) e^{-st} dt \\ &= \frac{\overline{A|v|^2}}{T} \int_0^{\infty} \frac{(2\alpha\beta)^2 e^{-(1-\alpha+sT)t/T}}{[(\alpha^2 + \beta^2) \sinh \beta t/T + 2\alpha\beta \cosh \beta t/T]^2} dt \\ &= \frac{\overline{A|v|^2} (4\alpha\beta)^2}{(\beta + \alpha)^4} \int_0^{\infty} \frac{e^{-(1+sT-\alpha+2\beta)t/T}}{[1 - (\frac{\beta-\alpha}{\beta+\alpha})^2 e^{-2\beta t/T}]^2} dt \quad \text{B-3} \end{aligned}$$

(U) Now replace  $2\beta t/T$  by  $y$  to give

$$W(s) = \frac{A|v|^2(4\alpha\beta)^2}{2\beta(\beta+\alpha)^4} \int_0^{\infty} \frac{e^{-(1+sT-\alpha+2\beta)y/2\beta}}{\left[1 - \left(\frac{\beta-\alpha}{\beta+\alpha}\right)^2 e^{-y}\right]^2} dy \quad B-4$$

$$= \frac{Z(1-Z^2)}{\mu} F(2, \mu; \mu+1; Z^2) \quad B-5$$

$$\text{where } \mu = \frac{sT+1+2\beta-\alpha}{2\beta}$$

$$\text{and } Z = \frac{\beta-\alpha}{\beta+\alpha}$$

(U) Using the series expansion for  $F(\quad)$ , equation B-5 becomes,

$$\begin{aligned} W(s) &= \frac{Z(1-Z^2)}{\mu} \sum_{n=0}^{\infty} \frac{(2)_n (\mu)_n}{(\mu+1)_n n!} Z^{2n} \\ &= Z(1-Z^2) \sum_{n=0}^{\infty} \frac{n+1}{\mu+n} Z^{2n} \quad B-6 \end{aligned}$$

(U) From this and the definition of  $\mu$  we see that the poles of  $W(s)$  lie at

$$s = -\frac{1}{T} (2\beta n - \alpha + 2\beta + 1) \quad n = 0, 1, 2, 3, \dots$$

$$= \frac{1}{T} [1 + 2(n+1)\beta - \alpha]$$

B-7

(U) Equation B-6 has the form of a driving point impedance of an RC network. It is well-known that the singularities of such a network alternate between zero and poles on the negative real axis and that the singularity closest to the origin is a pole. From this we can infer that the frequency spectrum peaks at the origin, and has a 3 db bandwidth equal to or greater than the 3 db bandwidth of the pole closest to the origin. Therefore, the two-sided bandwidth of the spectrum of  $w(t)$  will be at least,

$$Bw \geq \frac{1}{\pi T} (1 + 2\beta - \alpha)$$

B-8

or since  $\beta \geq \alpha$

$$Bw \geq \frac{1}{\pi T} [1 + \alpha]$$

$$\geq B_J \quad \text{since } \alpha = \pi B_J T$$

B-9

This is the statement made in the text.